

Activity 2

Here are two questions from the June Pure 2 paper:

- 3. (i)** Use algebra to prove that for all real values of x

$$(x - 4)^2 \geq 2x - 9 \quad (3)$$

- (ii)** Show that the following statement is untrue.

$$2^n + 1 \text{ is a prime number for all values of } n, n \in \mathbb{N} \quad (1)$$

- 8. (i)** Find the exact solution of the equation

$$8^{2x+1} = 6$$

giving your answer in the form $a + b \log_2 3$, where a and b are constants to be found. **(4)**

- (ii)** Using the laws of logarithms, solve

$$\log_5(7 - 2y) = 2 \log_5(y + 1) - 1 \quad (5)$$

Use the mark scheme to mark the following answers:

A

(1)

i) $(x-4)(x-4)$ $x^2 - 4x - 4x + 16$
 $x^2 - 8x + 16$

$x^2 - 8x + 16 \geq 2x - 9$
 $x^2 - 10x + 25 \geq 0$
 $(x-5)^2 \geq 0$

$x = 5$

$5 \geq 0$

ii) $2^n + 1$

$2^1 + 1 = 3$ True

$2^2 + 1 = 5$ True

$2^3 + 1 = 9$ False

$2^4 + 1 = 17$ True

$2^5 + 1 = 33$ False

The statement is not true the equation doesn't result a prime number

B

$$i) \quad (x-4)(x-4) \geq 2x-9$$

$$x^2 - 4x - 4x + 16 \geq 2x - 9$$

$$x^2 - 8x + 16 \geq 2x - 9$$

$$-2x + 9, -2x + 9$$

$$x^2 - 10x + 25 \geq 0$$

$$\left(x - \frac{10}{2}\right)^2 - \left(\frac{-10}{2}\right)^2 + 25 \geq 0$$

$$(x-5)^2 - 25 + 25 \geq 0$$

$$(x-5)^2 \geq 0$$

$(x-5)^2$ gives all ways positive value for any number or gives zero if x is zero so all values will be greater or equal to zero.

B (cont)

ii) even = 6 ~~odd = 3~~ odd = 3

$$2^6 + 1 = 65$$

not
prime

$$2^3 + 1 = 9$$

not prime

C

~~1~~ i) $(x-4)(x-4) \geq 2x-9$

$$x^2 - 4x - 4x + 16$$

$$x^2 - 8x + 16 \geq 2x - 9$$

$-2x \qquad -2x$

$$x^2 - 10x + 25 \geq 0$$

$$(x-5)$$

$x=5$ is a real value

all are real roots

ii) $2^1 + 1 = 3$

$$2^2 + 1 = 5$$

when ^{the power} ↑ gives an even number

(+1) gives us the odd

power gives an even number

so the (+1) change it to

an prime number / odd number

D

8i)

$$2x + 1 \log_2 8 = \log_2 6$$

$$(2x + 1)3 = \log_2 6$$

$$6x + 3 = \log_2 6$$

8i) $(2^3)^{2x+1}$

~~$\log 8$~~

$\log_8 6 \cdot 2x + 1$

$$2x + 1 = \frac{\log 6}{\log 8}$$

$$2x + 1 \log_8 8 = \log_2 6$$

$$2x + 1 + 1 = \log_2 6$$

$$2x + 2 = \log_2 6$$

$$2$$

$$x + 1 = \log_2 3$$

$$\frac{\log(2 \times 3)}{\log(2^3)}$$

$$\frac{1 + \log_2 3}{\log_2 8}$$

$$\frac{1 + \log_2 3}{2}$$

$$\frac{1}{3} + \log_2 3$$

$$\boxed{\frac{1}{3} + \log_2 3}$$

$$a = \frac{1}{3} \quad b = 1$$

$$\text{ii) } \log_5 (7 - 2y) = 2 \log_5 (y + 1) - 1$$

$$\log_5 \left(\frac{7 - 2y}{(y + 1)^2} \right) = -1$$

$$(y + 1)(y + 1)$$

$$y^2 + 2y + 1$$

$$\frac{1}{5} = \frac{7 - 2y}{y^2 + 2y + 1}$$

$$\begin{array}{rcl} y^2 + 2y + 1 & = & 35 - 10y \\ & & -35 + 10y \\ & +10y & \end{array}$$

$$y^2 + 12y - 34 = 0$$

~~$$y = -6 \pm \sqrt{40}$$~~

$$y = -6 + \sqrt{70}$$

$$y = -6 - \sqrt{70}$$

E

$$\begin{aligned} \text{i) } (2^3)^{2x+1} &= \log 6 \\ 6x+3 &= \log 6 \\ 6x+3 &= \log 6 \end{aligned}$$

$$\begin{aligned} \text{ii) } \log_5(7-2y) &= 2\log_5(y+1) - 1 \\ \log_5(7-2y) - 2\log_5(y+1) &= -1 \\ \log_5 \frac{(y+1)^2}{7-2y} &= -1 \\ \frac{y^2 + 2y + 1}{7-2y} &= -1 \\ (7-2y) \cdot -1 &= y^2 + 2y + 1 \\ -7 + 2y &= y^2 + 2y + 1 \\ -7 + 2y &= y^2 + 2y + 1 \\ y^2 + 8 & \\ y^2 &= 8 \\ y &= 2\sqrt{2} = 2.83 \end{aligned}$$

F

i. $2^{3(2x+1)} = 6$ $2^{6x} \times 2^3 = 6$

~~2~~ ~~2~~ $2^{6x} = \frac{3}{4}$

~~6x \log 2 = \log \frac{3}{4}~~ $6x \log 2 = \log \frac{3}{4}$

~~6x = -0.415~~ $6x = -0.415$

~~x = -0.069~~ $x = -0.069$

ii. $\log_5(7-2y) - \log_5(y+1)^2 = -1$

$\log_5\left(\frac{7-2y}{(y+1)^2}\right) = -1$

$\frac{7-2y}{(y+1)^2} = \frac{1}{5}$

$35 - \log y = y^2 + 2y + 1$

$y^2 + 12y - 34 = 0$

~~y = -6 \pm \sqrt{70}~~ $y = -6 \pm \sqrt{70}$

$y = -6 + \sqrt{70}$

G

$$\begin{aligned}
 \text{i) } (8^2)^x * 8 &= 6 & 8 &= 2^3 \\
 (2^6)^x * 8 &= 6 \\
 (2^6)^x * 2^3 &= 6 \\
 2^{6x+3} &= 6 & \log_2 8 &= 3 \\
 \log_2 6 &= 6x+3 \\
 \log_2 2 + \log_2 3 &= 6x+3 \\
 \cancel{3\log_2} & \\
 1 + \log_2 3 &= 6x+3 \\
 \log_2 3 &= 6x+2 \\
 \log_2 3 - \frac{1}{3} & \\
 \hline
 6 & \\
 \frac{1}{6} \log_2 3 - \frac{1}{18} & \\
 -\frac{1}{18} + \frac{1}{6} \log_2 3 &
 \end{aligned}$$

$$\text{ii) } \log_5 (7-2y) - \log_5 (y+1)^2 = -1$$

$$\log_5 \left(\frac{7-2y}{y^2+2y+1} \right) = -1$$

$$5^{-1} = \frac{7-2y}{y^2+2y+1}$$

$$\frac{1}{5} y^2 + \frac{2}{5} y + \frac{1}{5} = 7-2y$$

$$\frac{1}{5} y^2 + \frac{12}{5} y - \frac{34}{5}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-\frac{12}{5} \pm \sqrt{\left(\frac{12}{5}\right)^2 - 4 \times \frac{1}{5} \times -\frac{34}{5}}}{2 \times \frac{1}{5}}$$

$$y = -6 + \sqrt{70} = 2.4$$

$$y = -6 - \sqrt{70} = -14.4 \quad \times$$

$$y = 2.4$$

I

- 5 120 children go on an activity holiday.
The ratio of the number of girls to the number of boys is 3:5

On Sunday, all the children either go sailing or go climbing.

$\frac{16}{25}$ of the boys go climbing.

Twice as many girls go sailing as go climbing.

Work out how many children go sailing on Sunday.

$$3:5 = 120$$

$$45:75 \quad B = \frac{16}{25} \text{ of } 75 = 48$$

$$G = 2:1 = 45 \quad B = \begin{array}{r} 75 \\ - 48 \\ \hline 27 \end{array}$$

$$G = \frac{45}{3}$$

$$G = 30:15$$

$$\begin{array}{r} 27 \\ + 30 \\ \hline 47 \end{array}$$

47

(Total for Question 5 is 6 marks)

J

- 5 120 children go on an activity holiday.
The ratio of the number of girls to the number of boys is 3:5

On Sunday, all the children either go sailing or go climbing.

$\frac{16}{25}$ of the boys go climbing.

Twice as many girls go sailing as go climbing.

Work out how many children go sailing on Sunday.

$$\text{girls} = 3$$

$$\text{boys} = 5$$

$$\text{boy} = \frac{5}{8} \times 120 = 75$$

$$\text{girl} = \frac{3}{5} \times 120 = 45$$

$$\text{boys climbing} = \frac{16}{25} \times 75 = 48$$

$$\text{girls} = 24$$

$$48 + 24 = 72$$

72

(Total for Question 5 is 6 marks)

K

- 5 120 children go on an activity holiday.
The ratio of the number of girls to the number of boys is 3:5

On Sunday, all the children either go sailing or go climbing.

$\frac{16}{25}$ of the boys go climbing.

Twice as many girls go sailing as go climbing.

Work out how many children go sailing on Sunday.

$$3+5=8 \rightarrow 120 \div 8 = 15$$

$$15 \times 3 = 45, 15 \times 5 = 75$$

$$75 \div 25 = 3 \rightarrow +16 = 19$$

$$75 - 19 = 56$$

~~$$3 \times 8 = 24$$~~

✓ Sailing (8)

~~$$\div 8 = 5$$~~

$$120 - 56 = 64$$

$$\downarrow \div 2 = 32 \rightarrow \frac{32}{2} = 16$$

↓
Sailing (6)

↓
Climbing (6)

$$32 + 56 = 88$$

88



Teaching and Learning